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Waveguide and Resonator Perturbation Techniques Measuring Chirality and Nonreciprocity Parameters Biisotropic Materials

Sergei A. Tretyakov and Ari J. Viitanen

***Abstract*—Waveguide and resonator perturbation techniques are considered for determining electromagnetic parameters of general biisotropic, or nonreciprocal chiral, materials. The biisotropic materials are the most general linear isotropic media, whose constitutive relations are governed by four complex material parameters. The material parameters of biisotropic media can be obtained through measuring the change in the propagation constant of waveguide modes or measuring the shift in the resonant frequency for resonators with perturbation techniques. To measure these parameters a method utilizing waveguides or cavity resonators with two degenerate modes is proposed.**

I. INTRODUCTION

Interest in biisotropic materials has been recently widely increasing since they offer some novel promising applications in microwave technology and radio engineering, like low-reflection coatings [1], elimination of crosspolarization in microwave lens antennas [2] and construction of a twist polarizer [3]. The most general isotropic materials are characterized by four material parameters, dielectric permittivity, magnetic permeability, chirality parameter and nonreciprocity parameter. The biisotropic media are, in general, nonreciprocal, and the reciprocal special case is usually called chiral media. In electromagnetics terms, biisotropic materials were formalized for the first time in 1948, when Tellegen invented gyrator, a nonreciprocal circuit element, and considered what kind of material is needed to manufacture it [4]. It has been known that the material parameters of a biisotropic medium are associated with a model of the medium possessing both electric and magnetic dipole moments, parallel or antiparallel to each other [5].

To be able to determine the medium parameters of the biisotropic material is of great importance for practical applications. This work applies to chiral media in practice and gives a method to determine the material parameters for the general, theoretically interesting biisotropic media. In this paper we use the perturbation theory for general waveguides and resonators with small biisotropic inclusions and propose possible cavity resonator measurement techniques and waveguide perturbation techniques for measuring both the chirality parameter and the nonreciprocity parameter simultaneously. Free-space techniques are used to measure the chirality parameter in chiral composites [6]–[8], and the theoretical basis for the retrieval of the material parameters is the theory of reflection and transmission in chiral slabs. An alternative measurement technique for nonreciprocity parameter measurements is given in [9]. An approach to determination of the chirality parameter by using corrugated waveguides with chiral inclusions is discussed in [10]. A cavity resonator method for measuring the nonreciprocity parameter has been suggested in [11].

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II. PERTURBATION THEORY

The biisotropic medium is characterized by linear and isotropic constitutive relations with four scalar parameters. These relations can be written (assuming the $e^{j\omega t}$ time dependence), in the form

$$\bar{D} = \epsilon\bar{E} + \xi\bar{H}, \qquad \bar{B} = \mu\bar{H} + \zeta\bar{E}, \tag{1}$$

where ϵ and μ are dielectric permittivity and magnetic permeability of the medium, respectively, and the parameters ξ and ζ characterize coupling between the electric and magnetic fields. It is convenient to write the coupling parameters ξ and ζ in the form [12]:

$$\xi = (\chi - j\kappa)\sqrt{\mu_o\epsilon_o}, \qquad \zeta = (\chi + j\kappa)\sqrt{\mu_o\epsilon_o}, \tag{2}$$

where ϵ_o and μ_o are the free-space permittivity and permeability, respectively, and the parameter κ describes the magnitude of chirality and χ the nonreciprocity of the medium.

A. Waveguide Perturbation

Consider a waveguide with an arbitrary cross-section and ideally conducting walls. Field vectors of propagating modes in the empty guide have $e^{-j\beta_o z}$ dependence on the longitudinal co-ordinate z. A biisotropic rod of a small cross-section positioned in the waveguide causes a small change $\Delta\beta$ in the propagation factor β_o which can be measured. The unperturbed electric and magnetic fields $\bar{E}_o$, $\bar{H}_o$ and the perturbed fields $\bar{E}$, $\bar{H}$ inside the waveguide satisfy the Maxwell equations, where the $e^{j\omega t}$ time dependency is assumed and ∇_t is the two-dimensional gradient operator in the transverse plane.

In the conventional way, by forming the expression [13]

$$\bar{H}_o^* \cdot \nabla_t \times \bar{E} + \bar{H} \cdot \nabla_t \times \bar{E}_o^* - \bar{E}_o^* \cdot \nabla_t \times \bar{H} - \bar{E} \cdot \nabla_t \times \bar{H}_o^*$$

and integrating this expression over the cross-section area S of the waveguide, the following relation for the change of the propagation factor $\Delta\beta = \beta - \beta_o$ can be obtained:

$$\Delta\beta = \frac{\omega}{P_o} \int_{\Delta S} [(\epsilon - \epsilon_o)\bar{E} \cdot \bar{E}_o^* + (\mu - \mu_o)\bar{H} \cdot \bar{H}_o^* + \xi \bar{H} \cdot \bar{E}_o^* + \zeta \bar{E} \cdot \bar{H}_o^*] \, dS. \quad (3)$$

Here ΔS is the cross-section area of the biisotropic rod, and the asterisk denotes complex conjugate vectors. The factor in the denominator represents the power propagating in the waveguide

$$P_o = \int_S (\bar{E}_o^* \times \bar{H}_o + \bar{E}_o \times \bar{H}_o^*) \cdot \bar{u}_z \, dS, \quad (4)$$

where it is assumed that we can perturbationally approximate $\bar{E} \approx \bar{E}_o$, $\bar{H} \approx \bar{H}_o$ when integrating over the waveguide cross-section area. The boundary term which appears during the integration around the boundary of the waveguide with ideal conductor walls vanishes. The two last terms in (3) give a first-order change in the propagation factor with respect to the nonreciprocity parameter χ and to the chirality parameter κ.

To determine the fields inside the biisotropic inclusion of a small circular cross-section we make use of the quasi-static approximation for the transverse fields inside the biisotropic rod set in an unperturbed field $\bar{E}_{to}$, $\bar{H}_{to}$ [14]–[16] shown at bottom of the next page. Inserting the quasi-static fields into the expression for the change of the propagation factor (3), we obtain (6), shown at the bottom of the page, where we have denoted $\Delta_w = (\mu_r + 1)(\epsilon_r + 1) - (\chi^2 + \kappa^2)$ and the index t denotes the transverse field components. It is clearly seen that, by using small perturbation approximation to achieve the first-order change in the propagation factor due to the nonreciprocity factor χ, the unperturbed field configuration for the propagating mode must have a non-zero value for $Re(\bar{E}_o \cdot \bar{H}_o^*)$ [11], while the first-order change due to the chirality parameter κ we have when the unperturbed fields have a non-zero value for $Im(\bar{E}_o \cdot \bar{H}_o^*)$ [17]–[19]. We will see in the next subsection that this is also valid when using cavity resonator methods for determining material parameters.

B. Resonator Perturbation

A perturbation formula expressed in terms of electric and magnetic dipole moments of the inclusion has been published in [20]. Perturbation theory for cavity resonators with ideally conducting walls and small biisotropic inclusions was developed in [11]. The resonant frequency shift due to a biisotropic inclusion is given by the following exact formula

$$\Delta\omega = -\frac{\omega}{4W_o} \int_{\Delta V} [(\epsilon - \epsilon_o)\bar{E} \cdot \bar{E}_o^* + (\mu - \mu_o)\bar{H} \cdot \bar{H}_o^* + \xi \bar{H} \cdot \bar{E}_o^* + \zeta \bar{E} \cdot \bar{H}_o^*] \, dV \quad (7)$$

where $\bar{E}$ and $\bar{H}$ are the fields inside the sample, and $\bar{E}_o$ and $\bar{H}_o$ stand for the unperturbed fields. The integral is calculated over the inclusion volume ΔV only. If the perturbation inclusion is so small that we can replace the fields by their unperturbed values when calculating the integral over the cavity volume, then in the denominator W_o is proportional to the field energy in the unperturbed resonator:

$$W_o = \frac{1}{4} \int_V (\epsilon_o \bar{E}_o \cdot \bar{E}_o^* + \mu_o \bar{H}_o \cdot \bar{H}_o^*) dV. \quad (8)$$

For a special case of a spherical biisotropic sample we can use the quasistatic approximation for the fields inside a biisotropic sphere, which gives the following relations between the fields inside the sample and the external fields [16], [21]:

$$\begin{pmatrix} \bar{E} \\ \bar{H} \end{pmatrix} = \frac{3}{\Delta_r} \begin{pmatrix} \mu_r + 2 & -(\chi - j\kappa)\sqrt{\frac{\mu_o}{\epsilon_o}} \\ -(\chi + j\kappa)\sqrt{\frac{\epsilon_o}{\mu_o}} & \epsilon_r + 2 \end{pmatrix} \begin{pmatrix} \bar{E}_o \\ \bar{H}_o \end{pmatrix}, \quad (9)$$

with $\Delta_r = (\mu_r + 2)(\epsilon_r + 2) - (\chi^2 + \kappa^2)$. Substituting (9) into (7) leads to the expression for the shift of the resonance frequency

$$\Delta\omega = -\frac{3\omega}{4W_o\Delta_r} \int_{\Delta V} \Big[\epsilon_o[(\epsilon_r - 1)(\mu_r + 2) - (\chi^2 + \kappa^2)]\bar{E}_o \cdot \bar{E}_o^* + \mu_o[(\mu_r - 1)(\epsilon_r + 2) - (\chi^2 + \kappa^2)]\bar{H}_o \cdot \bar{H}_o^* + 6\chi\sqrt{\epsilon_o\mu_o} Re(\bar{E}_o \cdot \bar{H}_o^*) - 6\kappa\sqrt{\epsilon_o\mu_o} Im(\bar{E}_o \cdot \bar{H}_o^*) \Big] dV. \quad (10)$$

III. Chirality and Non-Reciprocity Parameter Measurements

Based on the perturbation analysis of the previous section, we can now study possibilities to measure the material parameters by waveguide or resonator techniques. As is seen from (6), (10), to have a first-order effect on the waveguide propagation factor or on the resonance frequency of a cavity proportional to the chirality parameter, we should have such a mode that $Im(\bar{E}_o \cdot \bar{H}_o^*)$ is non-zero. In contrast, to have a first-order effect due to the non-reciprocity parameter, we must have non-zero real part of the dot product $\bar{E}_o \cdot \bar{H}_o^*$. In this section we consider possible ways of achieving desired properties of the modes. Obviously, the modes must be degenerate, since otherwise the dot product $\bar{E}_o \cdot \bar{H}_o^*$ always vanishes due to the orthogonality of the modes. The two modes are degenerate if they have different field patterns but the same eigenfrequencies [22]. In the following we consider for simplicity the circular cross-section waveguides and cylindrical resonators. However, the theory can be applied to the general case of arbitrary cross-section guides by substituting corresponding solutions for Hertz vectors.

$$\begin{pmatrix} \bar{E}_t \\ \bar{H}_t \end{pmatrix} = \frac{2}{(\mu_r + 1)(\epsilon_r + 1) - (\chi^2 + \kappa^2)} \begin{pmatrix} \mu_r + 1 & -(\chi - j\kappa)\sqrt{\frac{\mu_o}{\epsilon_o}} \\ -(\chi + j\kappa)\sqrt{\frac{\epsilon_o}{\mu_o}} & \epsilon_r + 1 \end{pmatrix} \begin{pmatrix} \bar{E}_{to} \\ \bar{H}_{to} \end{pmatrix}. \quad (5)$$

$$\Delta\beta = \frac{\omega}{P_o} \int_{\Delta S} \Big(\epsilon_o \Big[\frac{2}{\Delta_w}[(\epsilon_r - 1)(\mu_r + 1) - (\chi^2 + \kappa^2)]\bar{E}_{to} \cdot \bar{E}_{to}^* + (\epsilon_r - 1)E_{zo}E_{zo}^* \Big] + \mu_o \Big[\frac{2}{\Delta_w}[(\mu_r - 1)(\epsilon_r + 1) - (\chi^2 + \kappa^2)]\bar{H}_{to} \cdot \bar{H}_{to}^* + (\mu_r - 1)H_{zo}H_{zo}^* \Big] + \chi\sqrt{\mu_o\epsilon_o} \Big[\frac{8}{\Delta_w} Re(\bar{E}_{to} \cdot \bar{H}_{to}^*) + 2Re(E_{zo}H_{zo}^*) \Big] - \kappa\sqrt{\mu_o\epsilon_o} \Big[\frac{8}{\Delta_w} Im(\bar{E}_{to} \cdot \bar{H}_{to}^*) + 2Im(E_{zo}H_{zo}^*) \Big] \Big) dS, \quad (6)$$

A. Waveguide Techniques

Let us consider the dot product $\bar{E}_o \cdot \bar{H}_o^*$ for eigenwaves in a waveguide with a circular cross-section of the radius a and assume ideally conducting walls. For H-modes, for example, in cylindrical co-ordinate system the dot product is

$$\bar{E}_o \cdot \bar{H}_o^* = \frac{\omega\mu_o\beta_o}{\rho}\left(\frac{\partial \Pi}{\partial \varphi}\frac{\partial \Pi^*}{\partial \rho} - \frac{\partial \Pi}{\partial \rho}\frac{\partial \Pi^*}{\partial \varphi}\right) = \frac{j2\omega\mu_o\beta_o}{\rho} Im(\frac{\partial \Pi}{\partial \varphi}\frac{\partial \Pi^*}{\partial \rho}) \qquad (11)$$

where Π is the Hertz potential function [22]. First, we see that the dot product $\bar{E}_o \cdot \bar{H}_o^*$ is always imaginary. Consequently, while using any waveguide with only H-modes, one can measure chirality but not the nonreciprocity parameter. In analogy, the same result can be stated for the E-modes. Moreover, to have the dot product non-zero and to provide non-zero coupling, we must have a degenerate mode. Indeed, substituting Π for circular waveguides in the form

$$\Pi(\rho,\varphi) = J_m(k_c\rho)[A\cos m\varphi + B\sin m\varphi] \qquad (12)$$

where $J_m(x)$ is the Bessel function of the first kind, $k_c = p'_{mn}/a$ and A, B the amplitude coefficients, leads to $\bar{E}_o \cdot \bar{H}_o^* \sim Im(AB^*)$. If either A or B is zero, the dot product vanishes. For E-modes, of course, one will get a similar result.

Let us see now if we can utilize two degenerate propagating modes of different types and consider the case when one of the modes is an E-mode and another is an H-mode. For example, in a circular waveguide modes E_{1n} and H_{0n} are degenerate, because $k_c = p'_{1n}/a = p_{0n}/a$ [22]. The dot product of the total fields reads

$$\bar{E}_o \cdot \bar{H}_o^* = (\bar{E}_{1n} \cdot \bar{H}_{1n}^* + \bar{E}_{0n} \cdot \bar{H}_{0n}^*) + (\bar{E}_{1n} \cdot \bar{H}_{0n}^* + \bar{E}_{0n} \cdot \bar{H}_{1n}^*) \qquad (13)$$

where the indices mark the fields of the corresponding modes. The first term vanishes because of the orthogonality of the the fields in each of the modes, but the latter term remains. Writing the dot product in terms of longitudinal parts of Hertz potentials Π for the H-mode and M for the E-mode

$$\bar{E}_o \cdot \bar{H}_o^* = k_c^4 M\Pi^* + \beta_o^2\left(\frac{\partial M}{\partial \rho}\frac{\partial \Pi^*}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial M}{\partial \varphi}\frac{\partial \Pi^*}{\partial \varphi}\right) - k^2\left(\frac{\partial M^*}{\partial \rho}\frac{\partial \Pi}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial M^*}{\partial \varphi}\frac{\partial \Pi}{\partial \varphi}\right), \qquad (14)$$

we see that this function is in general complex and we can have both real and imaginary coupling terms in (6).

Specifically, for a circular waveguide with E_{11} and H_{01} modes, we substitute the Hertz potentials

$$M(\rho,\varphi) = A\, J_1(k_c\rho)\cos\varphi, \Pi(\rho,\varphi) = B\, J_0(k_c\rho), \qquad (15)$$

where $k_c = 3.832/a$ and the free-space wave number $k = \omega\sqrt{\mu_o\epsilon_o}$. The result is

$$\bar{E}_o \cdot \bar{H}_o^* = k_c^2\cos\varphi\big[AB^*\big(k_c^2 J_0(k_c\rho)J_1(k_c\rho) + \beta_o^2 J_0'(k_c\rho)J_1'(k_c\rho)\big) - A^*B\, k^2 J_0'(k_c\rho)J_1'(k_c\rho)\big]. \qquad (16)$$

When the Hertz potentials of two degenerate modes are in phase (e.g., $A = B = 1$), the dot product is a real quantity. This allows one to measure non-reciprocity, since there is a first-order perturbation of the propagation factor, proportional to the nonreciprocity parameter. When the modes are 90 degrees out of phase ($A = 1, B = j$) we have imaginary coupling co-efficient, and the perturbation is proportional to the chirality parameter. (The last case was also considered in [19]).

B. Resonator Techniques

Let us next study the resonator problem by considering, for example, the cylindrical resonator with ideally conducting walls. The length of the resonator is L and the radius is a. Again, by taking two, for example, H-modes, and calculating $\bar{E}_o \cdot \bar{H}_o^*$, one arrives to

$$\bar{E}_o \cdot \bar{H}_o^* \sim Im(\frac{\partial \Pi}{\partial \varphi}\frac{\partial \Pi^*}{\partial \rho}), \qquad (17)$$

which is always a real function, in contrast with that for the waveguides, and, hence, the chirality parameter κ is not measurable. This could have been expected, since a resonator mode is a combination of two waveguide modes, propagating in the opposite directions. Perturbations due to chirality cancel out because of the isotropy and the reciprocity. However, we can measure the nonreciprocity parameter χ by exciting two degenerate H- (or two E-) modes.

Let us consider two degenerate modes of different types. In a cylindrical resonator of a circular cross-section, for example, the modes E_{1np} and H_{0np} are degenerate. The index $p = 1, 2, 3, \ldots$. After some calculation we find (18), shown at bottom of this page. In general, the dot product (18) is complex. More specifically, for a circular cross-section, let us consider the modes E_{111} and H_{011}. Substituting the Hertz potentials

$$M(\rho,\varphi) = A\, J_1(k_c\rho)\cos\varphi, \qquad \Pi(\rho,\varphi) = B\, J_0(k_c\rho), \qquad (19)$$

with $k_c = 3.832/a$, in the expressions of the fields results in (20), shown at bottom of this page. Thus, depending on the phase shift between the modes, the dot product (20) can be either real or imaginary. It is interesting to note that in a rectangular resonator $\bar{E}_o \cdot \bar{H}_o^*$ is always real [11].

As is seen, when the dot product (20) is a real quantity the chirality parameter κ cancels out from (10). This gives a way for measuring the nonreciprocity parameter, because the last two terms in (10) become proportional to χ. On the other hand, if the dot product is an imaginary quantity, the nonreciprocity parameter χ cancels out from (10), thus, giving a way for measuring the chirality parameter. However, in order to determine the Tellegen parameter χ and the chirality parameter κ, we still have to measure first μ and ϵ, for example, by using samples, the form of which are needle or disc and locate them properly in a cavity resonator [11]. Also the quantities Δ_w and Δ_r in the denominator of (6) and (10) have to be measured, for example, through the technique discussed previously in [11].

$$\bar{E}_o \cdot \bar{H}_o^* = \frac{1}{2}\sin\frac{2p\pi z}{L}\left[k_c^4 M\Pi^* - \left(\frac{p\pi}{L}\right)^2\left(\frac{\partial M}{\partial \rho}\frac{\partial \Pi^*}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial M}{\partial \varphi}\frac{\partial \Pi^*}{\partial \varphi}\right) - k^2\left(\frac{\partial M^*}{\partial \rho}\frac{\partial \Pi}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial M^*}{\partial \varphi}\frac{\partial \Pi}{\partial \varphi}\right)\right] \qquad (18)$$

$$\bar{E}_o \cdot \bar{H}_o^* = \frac{k_c^2}{2}\sin\frac{2\pi z}{L}\cos\varphi\left[AB^*\left(k_c^2 J_0(k_c\rho)J_1(k_c\rho) - \frac{\pi^2}{L^2}J_0'(k_c\rho)J_1'(k_c\rho)\right) - A^*Bk^2 J_0'(k_c\rho)J_1'(k_c\rho)\right]. \qquad (20)$$

IV. Conclusion

The coupling terms in the perturbation formulas for both the waveguide and resonator perturbations are proportional to the coupling parameters χ and κ in the constitutive equations, and to the dot product of the electric and magnetic fields in unperturbed modes. In the present study we have demonstrated that by exciting properly two degenerate modes in waveguides or resonators with ideally conducting walls, the perturbation is proportional to either the nonreciprocity parameter χ or the chirality parameter κ. This gives a way to measure the material parameters χ and κ separately. Moreover, it appears that it is possible to distinguish between the effects of chirality and nonreciprocity by changing the phase shift of the two modes in a waveguide or a resonator. This makes the method rather convenient and simple.

If there exist only H- or E-polarized fields in a waveguide, the coupling term is always imaginary, and the nonreciprocity of inclusions gives only a second-order effect on the propagation factor. In contrast, in a resonator with either H- or E-modes, the coupling term is always real, and there are no first-order effects on the resonant frequency due to the chirality parameter κ. Physically, the perturbational methods of the biisotropic media parameters measurement are based on the coupling between two orthogonal modes when a biisotropic sample is present inside a waveguide or a resonator. Since the coupling effect is small, it seems preferable to excite both the modes by an external source and to measure the shift of the resonant frequency or the propagation factor of a degenerate mode. This approach makes the effect more pronounced. The paper presents a theoretical treatment of the measurement problem and there are many practical considerations to be taken into account. For example, the two degenerate modes are also coupled because of the losses in the walls of a waveguide or a resonator, and that can mask small effects due to the inclusion. These and other possible sources for errors have to be eliminated in practical applications of the suggested measurement techniques.

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An Efficient Method for Computing the Capacitance Matrix of Multiconductor Interconnects in Very High-Speed Integrated Circuit Systems

Shui-Ping Luo and Zheng-Fan Li

***Abstract*— A new method for computing the capacitance matrix of multiconductor interconnects with finite metallization thickness is developed. Converting the vertical wall of the rectangular conductors into the equivalent horizontal strips allows the Green's function in the spectral domain and the FFT algorithm to be used, which makes the method more effective for computing capacitance matrix of the interconnects.**

I. Introduction

As is well known, the computation of the electrical parameters of the interconnects for very high-speed integrated circuit systems is a hard task, even with the quasi-TEM assumption. For such structures (for example, the chip-to-chip or on-chip interconnects for VHSIC), multiconductor transmission lines with finite metallization thickness in multilayered dielectric media are used. C. Wei, R. F. Harrington, and others, [1], [2] have employed the well known moment method using the total charge Green's function, which has a very simple form (like the free space Green's function). But the method is consuming

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